The Journal of FINANCE
The Journal of THE AMERICAN FINANCE ASSOCIATION

THE JOURNAL OF FINANCE • VOL. LXXVII, NO. 4 • AUGUST 2022

Quantifying Reduced-Form Evidence on Collateral Constraints

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ABSTRACT

This paper quantifies the aggregate effects of financing constraints. We start from a standard dynamic investment model with collateral constraints. In contrast to the existing quantitative literature, our estimation does not target the mean leverage ratio to identify the scope of financing frictions. Instead, we use a reduced-form coefficient from the recent corporate finance literature that connects exogenous debt capacity shocks to corporate investment. Relative to a frictionless benchmark, collateral constraints induce losses of 7.1% for output and 1.4% for total factor productivity (TFP) (misallocation). We show these estimated losses tend to be more robust to misspecification than estimates obtained by targeting leverage.

AN ACCUMULATING BODY OF EVIDENCE shows the causal effect of financing frictions on *firm-level* outcomes. For instance, Lamont (1997) shows that a reduction in oil prices leads nonoil subsidiaries of oil companies to reduce capital expenditures, Rauh (2006) exploits nonlinear funding rules for defined benefit pension plans to identify the role of internal resources on corporate investment, Gan (2007) and Chaney, Sraer, and Thesmar (2012) use variation in local house prices as shocks to firms' collateral value and show that collateral values affect investment, Chodorow-Reich (2014) combines the default of Lehman

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DOI: 10.1111/jofi.13158

Brothers with the stickiness of banking relationships to show how bank lending frictions distort labor demand, and Lian and Ma (2021) and Greenwald (2019) show that exogenously binding covenants distort investment.[1] While this literature safely rejects the null hypothesis that firms are not financially constrained, it provides little guidance on the economic importance of financial constraints that derive from their "well-identified" estimates. The objective of this paper is to help fill this gap.

To do so, we structurally estimate a general equilibrium model of investment with financial frictions. Among the set of targeted moments, we match one of the well-identified estimates that helps to identify the quantitative importance of financial frictions. Simple counterfactual analyses reveal substantial aggregate costs of financing constraints. In our baseline estimation, we find that relative to a benchmark with no financial frictions, collateral constraints induce aggregate output losses of 7.1% and total factor productivity (TFP) losses of 1.4% due to misallocation. This quantification exercise, which uses a structural approach to interpret well-identified reduced-form evidence on financing frictions, is the paper's first contribution to the literature.

Our inference contrasts with the standard approach, which typically identifies the importance of financial frictions by targeting the average debt-to-capital ratio (leverage). We conduct an in-depth analysis of the robustness of our inference relative to this standard approach. More specifically, we consider robustness to model misspecification, that is, to forces that may be present in the data but that do not feature in our model. We develop a simple approach to analyze nonlocal misspecification errors in structural work. Our methodology is analogous to "robustness checks" in reduced-form analysis, where researchers incorporate alternative interpretations into their (empirical) model and verify that this does not affect their findings. Similarly, we consider a large number of alternative models and propose a systematic methodology to measure the size of misspecification bias that arises under these alternatives. This method is intuitive and computationally fast. By considering alternative interpretations of the data, it helps mitigate the concerns about the importance of particular economic mechanisms that are left out of the structural model. This systematic approach to evaluating robustness to misspecification is the paper's second contribution.

We now turn to a more detailed description of our empirical approach. We focus on a pervasive source of financing friction, namely, collateral constraints. Our parsimonious structural model adds a collateral constraint and costly equity issuance to a standard neoclassical model of investment with adjustment costs (Jorgenson (1963), Lucas (1967), Hayashi (1982)).[2] Physical capital and real estate can be used as collateral. Real estate prices fluctuate randomly.

[1] Other contributions include, among others, Banerjee and Duflo (2014), Lemmon and Roberts (2010), Faulkender and Petersen (2012), Zia (2008), Zwick and Mahon (2017), Benmelech, Bergman, and Seru (2011), Benmelech, Frydman, and Papanikolaou (2019).

[2] While we do not explicitly provide a microfoundation for the collateral constraint, it arises naturally from limited enforcement models as in Hart and Moore (1994).

An increase in real estate price increases firms' collateral value, leading to higher debt capacity and increased investment. This mechanism echoes the empirical findings in Gan (2007) and Chaney, Sraer, and Thesmar (2012), who exploit variation in real estate prices to estimate the sensitivity of firm-level investment to collateral values. This estimate, which we denote by β throughout the paper, is the key moment we target in our structural estimation to identify the importance of collateral constraints. Our estimation reveals that firms can pledge only about 25% of their capital stock. In the cross section, we find that constrained firms are more likely to be growth firms and low-productivity firms. Consistent with the literature on misallocation (Hsieh and Klenow (2009)), constrained firms have a high marginal revenue product of capital (MRPK) and a high Tobin's Q.

We next aggregate our findings to evaluate the importance of financing constraints in general equilibrium. The estimated model is nested into a simple equilibrium framework in which firms compete for customers, workers, and capital goods. We compare the steady state of the estimated economy with a counterfactual economy in which firms can obtain frictionless financing.[3] Relative to the frictionless benchmark, aggregate output is about 7% smaller in the estimated economy, of which 1.4% is due to lower TFP, that is, misallocation of labor and capital across firms (Hsieh and Klenow (2009), Midrigan and Xu (2014), Moll (2014)). The larger driver of this aggregate output loss, however, is lower factor use. In the estimated economy, the aggregate capital stock is 13.7% lower than that in the frictionless benchmark. Similarly, employment is 2.4% lower.[4]

While we are not the first to measure the aggregate costs of financial constraints, our paper differs from earlier studies in an important way: we identify the extent of financing constraints by targeting a moment that the empirical corporate finance literature directly ties to financing constraints. In the macrofinance literature, financial frictions are typically calibrated instead by targeting a moment related to corporate leverage in the economy. Intuitively, high corporate leverage should imply that firms can easily pledge capital to lenders and that financial constraints are loose. A large share of the macroeconomic literature uses *aggregate* data and target either an aggregate debt-to-capital ratio (e.g., Bernanke, Gertler, and Gilchrist (1999), Jermann and Quadrini (2012), Kahn and Thomas (2013), Liu, Wang, and Zha (2013), and Jo and Senga (2019)) or an aggregate debt/external finance-to-output ratio (e.g., Amaral and Quintin (2010), Buera and Shin (2013), Midrigan and Xu

[3] Of course, a counterfactual in which there are no financing constraints is certainly not policy-relevant, but it serves as a useful benchmark to measure the extent to which financing constraints are binding.

[4] In line with the macroeconomic literature, we formally quantify the cost of financing frictions, but not their potential benefit. We model collateral constraints in a reduced-form way and do not take a stance on whether the rationale behind these collateral constraints is efficient in a second-best sense.

(2014), Moll (2014), and Itskhoki and Moll (2019)).[5] Other papers in macroeconomics use firm-level data and focus on firms' average leverage ratio (e.g., Cooley and Quadrini (2001), Arellano, Bai, and Zhang (2012), Ottonello and Winberry (2020)) or a cross-sectional relationship between leverage and capital (e.g., Garcia-Macia (2017), Gopinath et al. (2017)).[6]

The structural corporate finance literature shares this focus on firm-level data. Papers typically target the average leverage ratio among Compustat firms to estimate either the pleadgability of physical assets (e.g., Hennessy and Whited (2005), Li, Whited, and Wu (2016), Nikolov, Schmid, and Steri (2019)) or the deadweight loss that firms experience in bankruptcy (Hennessy and Whited (2007), Gomes and Schmid (2010), Michaels, Beau, and Whited (2018)). This reliance on leverage as a measure of financial friction stands in contrast to the empirical corporate finance literature, which instead relies on the estimation of firms' response to plausibly exogenous variation in financial constraints (e.g., Lamont (1997), Rauh (2006), or Chaney, Sraer, and Thesmar (2012)). Our paper bridges the gap between the quantitative and reduced-form empirical literature on financing constraints by drawing the quantitative implications of such reduced-form estimates.[7]

What do we gain by relying on a "well-identified" moment as opposed to leverage? In Section V, we compare the two identification strategies in terms of their robustness to misspecification. In doing so, we propose a new and simple methodology to explore robustness to nonlocal misspecification in structural work. Similar to standard robustness checks in reduced-form research, we augment our baseline approach by incorporating alternative mechanisms and examining how this affects inference.

Our initial step follows the methodology developed by Andrews, Gentzkow, and Shapiro (2017). We compute the local sensitivity of output and TFP losses to both average leverage and β. The sensitivity matrix is a generalized inverse of the Jacobian matrix. The sensitivity to leverage is twice as large as the sensitivity to β. This diagnostic suggests that estimates targeting leverage are potentially more exposed to misspecification bias (Andrews, Gentzkow, and Shapiro (2017)). However, this evidence is only suggestive. Actual misspecification bias also depends on the extent to which leverage and β are themselves affected by misspecification. We investigate the effect of misspecification on these moments in two ways.

First, we consider sources of misspecification that arise purely from measurement issues. We assume that our model is correctly specified but that the moments we use for estimation are mismeasured. We consider several

[5] Greenwood, Sanchez, and Wang (2013) also use aggregate data, but their calibration relies on the average intermediation spread.

[6] Gilchrist, Sim, and Zakrajšek (2014) rely instead on the median of the BBB-Treasury spread in the United States.

[7] In Section I of the Internet Appendix, we offer a detailed review of the quantitative literature in macroeconomics and corporate finance that focuses on measuring financial frictions. We put a special emphasis on the moments targeted in the estimation/calibration and the resulting parameter estimates. The Internet Appendix may be found in the online verison of this article.

sources of mismeasurement highlighted in the corporate finance literature. For instance, the capital stock may be mismeasured because of operating leases (Rampini and Eisfeldt (2009)), or intangibles (Peters and Taylor (2017)), or because economic depreciation is smaller than accounting depreciation. Similarly, secured debt may be mismeasured in the presence of operating leases (Rampini and Eisfeldt (2009)), account payables (Barrot (2016)), or unsecured debt (Benmelech, Kumar, and Rajan (2020)). We estimate average leverage and β in our sample using six alternative measures of debt and capital that account for such measurement issues. We find that average leverage is typically more sensitive to these alternative measures than β. Combined with the lower sensitivity of TFP and output losses to β, this finding suggests that estimations targeting leverage may result in higher misspecification bias than estimations targeting β.

Second, we consider cases in which the model itself is misspecified. We simulate a large number of data sets generated by alternative models that deviate from our baseline model along several dimensions, including the presence of intangible capital, unobserved debt capacity, or unsecured debt capacity. Because we consider potentially large deviations, we depart from Andrews, Gentzkow, and Shapiro (2017) and adopt a global approach to measuring misspecification bias. More specifically, we estimate our baseline (misspecified) model using the simulated data sets in two ways: our first estimation targets leverage, while our second estimation targets the reduced-form coefficient β. Since we know the data-generating process, we can directly measure misspecification bias for each of these estimations by comparing the estimated TFP or output loss to their true value. Our analysis of 4,000 alternative models reveals that estimates obtained by targeting β suffer less from misspecification bias than those obtained by targeting leverage. Of course, this approach requires that we "specify the misspecification," as with typical robustness checks used in reduced-form research.

Our misspecification analysis explores a large number of alternative models, 4,000 in our application. In principle, running 4,000 estimations using simulated method of moments (SMMs) on simulated data sets would be computationally intensive. We avoid this computational burden by developing a simple technique for such Monte Carlo (MC) experiments. We start by estimating the baseline model once on the actual data. We next simulate the model using alternative parameters drawn in a relatively large space around the estimated parameters. In particular, for each set of parameters, we compute the simulated moments. We can then estimate, on the generated data sets of parameters and corresponding moments, a nonparametric function that links moments to parameters. We show that this relationship between moments and parameters is tightly estimated. We can therefore use this relationship to estimate the baseline model on the 4,000 data sets generated by the alternative models we consider when analyzing misspecification (e.g., models that feature unsecured debt, intangible capital, or other ingredients that do not feature in our baseline model).

The function mapping parameters to moments makes it computationally inexpensive to recover parameters from a large set of moments generated by alternative misspecified models. We bypass the need for more than a single SMM estimation. We believe this approach is useful for robustness checks in structural estimation, allowing one to explore sources of potential misspecification.

The paper is organized as follows. Section I calculates the key moments used in our inference on financial constraints. Section II presents our formal model of firm dynamics with collateral constraints. Section III structurally estimates the model using U.S. firm-level data. Section IV describes and implements the general equilibrium analysis and our counterfactual measure of the aggregate effects of collateral constraints. Section V compares misspecification bias that arises in estimations that target leverage versus the reduced-form coefficient β. Finally, Section VI concludes.

I. Real Estate Collateral and Investment

In this paper, we base our estimation on a measure of financing constraints coming from the reduced-form literature. We estimate the effect of real estate collateral on investment as in Chaney, Sraer, and Thesmar (2012). Construction of the data is detailed in that paper. The data set is a panel of publicly listed firms from 1993 to 2006 extracted from Compustat. We require that these firms supply information about the accounting value and cumulative depreciation of land and buildings (items ppenb, ppenli, dpacb, dpacli) in 1993. We combine this information with office prices in the metropolitan statistical area (MSA) in which headquarters are located to obtain a measure of the market value of firms' real estate holdings normalized by the previous year's property, plant, and equipment (PPE). We label this measure for firm i at date t $REValue_{it}$.

We next follow the preferred specification of Chaney, Sraer, and Thesmar (2012) and run the regression

$$\frac{i_{it}}{k_{it-1}} = a + \beta . \frac{REValue_{it}}{k_{it-1}} + Offprice_{it} + controls_{it} + \nu_{it}, \tag{1}$$

where i_{it} is investment (item capx), k_{it-1} is the lagged stock of productive capital (item ppent), and $Offprice_{it}$ is an index for office prices in the MSA in which firm i's headquarters is located. This index is available from Global Real Analytics for 64 MSAs. We include the same controls as in their table V, column (5), that is, firm- and year-specific fixed effects, as well as firm-level controls interacted with real estate prices. We cluster error terms ν_{it} at the firm level. We are interested in the reduced-form moment β, the estimated impact of real estate value on investment.

The only difference with Chaney, Sraer, and Thesmar (2012) is that we add about 900 MSA $\times$ year fixed effects, which forces identification on the comparison between owners and renters to be within MSA-years. We report regression results in Table IA.I in the Internet Appendix. This additional control leaves the estimate of their table V, column (5), unchanged at $\beta = 0.06$. The t-statistic weakens somewhat but remains high at 6.1 in this highly saturated specification. This moment suggests that every \$1 of real estate appreciation translates into \$0.06 of additional investment. The rest of the paper quantifies the implication resulting from this reduced-form estimate in terms of firm-level financial friction and aggregate efficiency and output losses.

II. The Model

In this section, we lay out our model of investment dynamics under collateral constraints. The economy is populated by heterogeneous, financially constrained firms, which combine capital and labor to produce differentiated goods. Those differentiated goods are then combined into a final good, consumed by a representative consumer, and used as a capital good.

A. Production Technology and Demand

The firm-level model is close to Hennessy and Whited (2007): it includes a tax shield for debt and a cost of equity issuance. It is also close to Liu, Wang, and Zha (2013): firms face a collateral constraint. The firm's shareholder is risk-neutral and her time discount rate is r. Firm i produces output q_{it} combining capital k_{it} and efficiency units of labor l_{it} into a Cobb-Douglas production function with capital share α,

$$q_{it} = F\left(e^{z_{it}}, k_{it}, l_{it}\right) = e^{z_{it}}\left(k_{it}^{\alpha} l_{it}^{1-\alpha}\right), \tag{2}$$

with z_{it} the firm's log TFP following the AR(1) process:

$$z_{it} = \rho z_{it-1} + \eta_{it},$$

and σ^2 the variance of the innovation η_{it}. The firm faces a downward sloping demand curve with constant elasticity $\phi > 1$,

$$q_{it} = Q p_{it}^{-\phi}, \tag{3}$$

where Q is aggregate spending and will be determined in equilibrium (see Section IV).

Labor is fully flexible. The wage w is also determined in equilibrium. As labor is a static input, the total profits of the firm, net of labor input and before taxes, is given by

$$\pi\left(z_{it}; k_{it}\right) = \max_{l_{it}}\{p_{it} q_{it} - w l_{it}\} = bQ^{1-\theta} w^{-\frac{(1-\alpha)}{\alpha}\theta} e^{\frac{\theta}{\alpha} z_{it}} k_{it}^{\theta}, \tag{4}$$

with b a scaling constant and $\theta \equiv \frac{\alpha(\phi-1)}{1+\alpha(\phi-1)} < 1$.

B. Input Dynamics

While labor is a static input, capital is not. Capital accumulation is subject to depreciation, time to build, and adjustment costs. Gross investment i_{it} is given by

$$k_{it+1} = k_{it} + i_{it} - \delta k_{it}, \tag{5}$$

where δ is the depreciation rate. In period t, investing i_{it} entails a convex cost of $\frac{c}{2}\frac{i_{it}^2}{k_{it}}$. In addition, in period t the firm pays for capital that will only be used in production in period $t+1$: this one-period time to build for capital is conventional in the macro literature (Hall (2004), Bloom (2009)) and acts as an additional adjustment cost. Introducing adjustment costs to capital is important in our estimation exercise because they generate patterns qualitatively similar to financing constraints and could therefore be a natural confounding factor in our estimation procedure. For instance, adjustment costs make capital vary less than firm output, which generates a natural dispersion in capital productivities, mimicking financing constraints (Asker, Collard-Wexler, and Loecker (2014)). As we will show below, using the reduced-form moments presented in Section I allows us to identify both frictions separately.

We do not, however, include fixed adjustment costs in our model, a choice also made by Gourio and Kashyap (2007). Our estimation targets firm-level data at an annual frequency, for which investment is not very lumpy. In our sample, only 4% of the observations have an investment rate smaller than 2% of capital.[8]

C. Financing Frictions and Capital Structure

The firm finances investment out of retained earnings, debt, and equity issuance to outside investors. We denote net debt by d_{it}, $d_{it} < 0$ means that the firm holds cash. As is standard in the structural corporate finance literature (Hennessy and Whited (2005)), we only consider short-term debt contracts with one-period maturity. Debt is risk-free and pays an interest rate r[9], which is determined in equilibrium in Section IV. For an amount d_{it} of debt issued at date t, the firm commits to repay $(1+r)d_{it+1}$ at date $t+1$. Finally, the interest rate the firm receives on cash is lower than the interest rate it has to pay on its debt: if the firm has negative net debt, it receives a positive cash inflow of $-(1+(1-m)r)d_{it+1}$, where $0 < m < 1$.

Consistent with the corporate finance literature, we also assume that firm profits net of interest payments and capital depreciation, δk_{it}, are taxed at rate τ. This tax rate applies to both negative and positive income so that firms

8 To compute the investment rate, we divide item capx by lagged item ppent.

9 While this risk-free interest rate could in principle be time-varying, it is constant in our model, pinned down by the consumer's Euler equation with no aggregate risk, and thus we omit the t subscript for simplicity.

receive a tax credit when their accounting profits are negative.[10] Other papers make alternative assumptions to make debt attractive to firms, assuming that debt holders are intrinsically more patient than shareholders or that shareholders with log utility seek to smooth consumption as in Midrigan and Xu (2014). Finally, note that all tax proceeds are rebated to the representative consumer (see Section IV).

The financing frictions come from the combination of two constraints. First, equity issuance is costly: if preissuance cash flows are x, cash flows net of issuance costs are given by

$$G(x) = x(1 + e\mathbf{1}_{x<0}),$$

where $e > 0$ parameterizes the cost of equity issuance. Second, firms face a collateral constraint, which arises from limited enforcement (Hart and Moore (1994)). We follow Liu, Wang, and Zha (2013) and adopt the following specification for the collateral constraint:

$$(1 + r)d_{it+1} \leq s\big((1 - \delta)k_{it+1} + \mathbb{E}[p_{t+1}|p_t] \times h\big). \tag{6}$$

The total collateral available to the creditor at the end of period $t + 1$ consists of depreciated productive capital $(1 - \delta)k_{it+1}$ and real estate assets with value $p_{t+1}h$. We assume that log p_t is a discretized AR(1) process. The share of the collateral value realized by creditors, s, captures the quality of debt enforcement, as well as the extent to which collateral can be redeployed and sold.[11]

In assuming that the quantity of real estate h is the same across firms and over time, we abstract from issues related to heterogeneity in real estate ownership. This is an important limitation of this paper. In reality, firms' decision to buy or lease real estate assets can depend on expected productivity, investment opportunities, local factor prices, and financing constraints. We leave analysis of how endogeneity of real estate ownership affects current investment decisions for future research and focus here on measuring and aggregating financial frictions given the observed levels of real estate ownership in the data.

D. The Optimization Problem

The firm is subject to a death shock with probability D, but infinitely lived otherwise. Every period, physical capital and debt are chosen optimally to

[10] As a result, debt is tax-free, which creates an incentive for firms to increase their leverage. This assumption marginally simplifies exposition and is consistent with several features of the tax code such as the presence of tax loss carryforwards, but is not crucial for our results.

[11] The formulation of the collateral using the expected future value of collateral is standard in macroeconomics. It can be justified as an optimal contract in a setup where (i) the firm has the entire bargaining power in its relationship with creditors, (ii) the firm cannot commit not to renegotiate the debt contract at the end of period t, and (iii) collateral can be seized only at the end of period $t + 1$.

maximize a discounted sum of per-period cash flows, subject to the financing constraint. The firm takes as given its productivity and local real estate prices, and forms rational expectations for future productivities and real estate prices.

Define by $V(S_{it}; X_{it})$ the value of the discounted sum of cash flows given the exogenous state variables $X_{it} = \{z_{it}, p_t\}$ and the past endogenous state variables $S_{it} = \{k_{it}, d_{it}\}$. Shareholders are assumed to be perfectly diversified, so their discount rate is the same as risk-free debt r.

This value function V is the solution to the Bellman equation

$$\begin{cases} V(S_{it}; X_{it}) & = \max\limits_{S_{it+1}} \Big\{ CF + \frac{1-D}{1+r} \mathbb{E}\big[V(S_{it+1}; X_{it+1})|X_{it}\big] + \frac{D}{1+r}\big(k_{it+1} - (1+\tilde{r}_{it})d_{it+1}\big)\Big\} \\ \text{s.t.} & (1+r)d_{it+1} \leq s\big((1-\delta)k_{it+1} + \mathbb{E}[p_{t+1}|p_t] \times h\big) \\ \text{with:} & CF = G\Big(\pi(z_{it}; k_{it}) - i_{it} - \frac{c}{2}\frac{i_{it}^2}{k_{it}} + d_{it+1} - (1+\tilde{r}_{it})d_{it} - \tau\big(\pi(z_{it}; k_{it}) - \tilde{r}d_{it} - \delta k_{it}\big)\Big) \\ & i_{it} = k_{it+1} - (1-\delta)k_{it} \\ & \tilde{r}_{it} = r \text{ if } d_{it} > 0 \text{ and } (1-m)r \text{ if } d_{it} \leq 0, \end{cases} \tag{7}$$

where the second term in the maximand ($\frac{D}{1+r}(k_{it+1} - (1+\tilde{r}_{it})d_{it+1})$) corresponds to the shareholder's payoff in the event of firm death. This term avoids a bias toward borrowing. If bankers could recover capital when a firm exits, shareholders would have an incentive to borrow more to transfer value from states of nature in which they cannot consume to states in which the firm survives. By assuming that shareholders receive the remaining capital when the firm exits, we ensure that this risk-shifting behavior does not drive the capital structure decisions of firms in our model.

Aggregate demand Q and the real wage w are equilibrium variables that the firms take as given when optimizing inputs. Given the absence of aggregate uncertainty and the steady-state assumption, they are fixed over time. Due to downward-sloping demand, firms have an optimal scale of production. A firm initially below this level accumulates capital, but only gradually because of convex adjustment costs and time to build. Finally, spending on adjusting capital is bounded by the collateral constraint. When the value of a firm's real estate assets increases, the collateral constraint is relaxed and the firm finances more of the cost of adjusting toward its desired scale. This generates the response of investment to shocks to collateral value documented in Section I.

III. Structural Estimation

A. Estimation Procedure

We estimate the key parameters of the model via SMM. The entire procedure is described in detail in Section II of the Internet Appendix. We look for the set of parameters $\hat{\boldsymbol{\Omega}}$ such that model-generated moments $\mathbf{m}(\hat{\boldsymbol{\Omega}})$ on simulated data fit a predetermined set of data moments $\mathbf{m}$. If we could solve the model analytically, we could just invert the system of equations given by model-based moments. Because our model does not have an analytic solution, we need to use indirect inference to perform the estimation. Such inference is done in two steps:

(i) For a given set of parameters, we solve the Bellman problem (7) numerically and obtain the policy function $S_{it+1} = (d_{it+1}, k_{it+1})$ as a function of $S_{it} = (d_{it}, k_{it})$ and exogenous variables $X_{it} = (z_{it}, p_t)$. We discretize the state space (S, X) into a grid that is as fine as possible to minimize numerical errors in the presence of hard financing constraints. This is critical: 1% to 2% numerically generated error would be too large to quantify aggregate effects of this order of magnitude. Solving the model repeatedly to estimate our structural parameters would not be feasible on a central processing unit (CPU) (several hours per iteration), so we use a graphics processing unit (GPU) instead (a few minutes per iteration), as described in Section II.A of the Internet Appendix.

(ii) Our parameter estimates minimize the distance from simulated to data moments,

$$\widehat{\Omega} = \arg\min_{\Omega} (m - \widehat{m}(\Omega))' W (m - \widehat{m}(\Omega)),$$

where the weighting matrix W is the inverse of the variance-covariance matrix of data moments. Standard errors are calculated by bootstrapping. Section II.B of the Internet Appendix describes how we escape the many local minima of our objective function and how we correct for both estimation and simulation errors.

B. Predefined and Estimated Parameters

The model has 15 parameters. We calibrate 10 of them using estimates from the literature or the data. We estimate the five remaining parameters.

Predefined parameters. Our 10 calibrated parameters are as follows. We set the capital share to $\alpha = 1/3$ following Bartelsman, Haltiwanger, and Scarpetta (2013) and demand elasticity to $\phi = 6.7$, which is within the range in Broda and Weinstein (2006) (18% markups in the absence of adjustment costs). Log real estate prices, $\log p_t$, follow a discretized AR(1) process. We estimate this AR(1) process on detrended logged real estate prices and find a persistence of 0.62 and innovation volatility of 0.06. Both AR(1) processes for $\log z_t$ and $\log p_t$ are discretized using Tauchen's method. The rate of obsolescence of capital is set to $\delta = 6\%$ as in Midrigan and Xu (2014). The risk-free borrowing rate r is fixed at 3%, while the lending rate is set to $(1 - m)r = 2\%$. We fix the death rate D to 8%, which corresponds to the turnover rate of firms in our data. We set the corporate tax rate τ to 33%. Since one may argue that effective tax rates may differ from statutory tax rates, below we explore the robustness of our inference with respect to the tax rate. The amount of real estate collateral h is set to match the average ratio of real estate to capital h/k_t exactly (0.14 for the average ratio of real estate, Compustat item land + building in 1993, to total assets, Compustat item at). Finally, we normalize $w = 0.03$ and $Q = 1$ for the estimation. This normalization is done without loss of generality because, in this model, the couple (Q, w) used for estimation has no effect on estimated structural parameters and aggregate outcomes (Sraer and Thesmar (2018)).[12] The parameters Q and w are endogenously determined in general equilibrium in our counterfactual analyses (see Section IV).

Estimated parameters. We estimate five parameters: the persistence ρ and innovation volatility σ of log productivity, the collateral parameter s, the adjustment cost c, and the cost of equity issuance e.

C. Data Moments

We compute the moments on the Compustat sample described in Section I. We describe them here with a short heuristic discussion on identification. In the next section, we discuss identification more systematically and show how simulated moments vary with parameters.

First, in the spirit of Midrigan and Xu (2014), we use the short- and long-term volatility of output to estimate the persistence and volatility of log sales. In our sample, the volatility of the change in log sales ($\log sales_{it} - \log sales_{it-1}$, Compustat item: sale) equals 0.327. The volatility of the five-year change in log sales ($\log sales_{it} - \log sales_{it-5}$) equals 0.912. The fact that five-year growth is less than five times more volatile than one-year growth contributes to the identification of the persistence parameter. Targeting these two moments instead of directly matching the persistence coefficient of log sales makes our estimation less sensitive to short-panel bias. Indeed, with firm fixed effects, even though in our panel firms are present about nine years on average, a fixed-effect estimator of persistence is strongly downward biased (Nickell (1981)). Targeting variances of log changes at various horizons allows us to bypass this problem.

Second, we use the autocorrelation of investment to identify quadratic adjustment costs (Bloom (2009)). For each firm in our panel we compute the ratio $\frac{i_{it}}{k_{it-1}}$ of capital expenditures (Compustat item: capx) to lagged capital stock (Compustat item: ppent). We then regress this ratio on firm fixed effects and extract the residuals. We next compute the autocorrelation of these residuals. This is done to filter out cross-sectional heterogeneity in ratios. The correlation between $\frac{i_{it}}{k_{it-1}}$ and $\frac{i_{it-1}}{k_{it-2}}$ in our data is 0.165. Adjustment costs compel the firm to smooth its investment policy in response to a productivity shock

[12] The intuition is as follows (Sraer and Thesmar (2018)). In partial equilibrium (i.e., for (Q, w) fixed), the level of (Q, w) scales up all firm outcomes by the same constant factor (call it φ). As a result, the simulated moments do *not* depend on the level of (Q, w) chosen. Ratios like leverage, investment rate, and real estate value divided by capital are unaffected when scaling by φ. The variance of log sales is also not affected by the scaling. This ensures that the set of structural parameters that we estimate via SMM does not depend on the level of (Q, w) chosen. Besides, aggregate outcomes are also insensitive to the level of (Q, w) chosen in estimation. In our macro framework, the constant elasticity of substitution (CES) aggregator that we (like many others in the macrofinance literature) use ensures that aggregate output and TFP depends only on the distribution of sales-to-capital ratio (MRPK), $\frac{py}{k}$. MRPKs are ratios and thus insensitive to the level of (Q, w) used to simulate the economy.

(Asker, Collard-Wexler, and Loecker (2014)). Financing frictions add to this smoothing motive.

Third, we use a direct measure of financing constraints to identify the collateral constraint parameter s, the sensitivity of investment to real estate value, which corresponds to the reduced-form moment β estimated from Equation (1). This regression coefficient is directly related to financing frictions: under our identifying assumption, this coefficient would be statistically insignificant absent financing frictions. While one can reject the absence of financing frictions if this coefficient is positive, its precise level does not map one for one into any structural parameter of our model. It does, however, allow us to identify the level of financing frictions through indirect inference. In Section V, we show that our inference based on β is more robust to misspecification than an inference that would be based on targeting a leverage ratio.

Fourth, we use data on equity issues to identify the cost of equity issuance (Hennessy and Whited (2007)). We compute the average ratio of net positive equity issuance to value-added, the relevant empirical counterpart for revenue $p_{it}q_{it}$ in the model. For each firm, we compute net equity issues as stock sales (item sstk) minus cash dividends (item dv) and share buybacks (item prstkc). We then take the maximum of this number and zero and normalize it by value-added. Since Compustat does not have a variable for value-added, we approximate value-added by 60% of total sales (item sale), assuming a 40% gross margin ratio as in Asker, Collard-Wexler, and Loecker (2014). The targeted moment corresponds to the average of this ratio across all firms in our sample, 0.026.

D. Parameter Identification

In this section, we discuss local identification of the parameters of the model. Specifically, we show the relationship between empirical moments and model parameters around the main SMM estimate for (s, c, ρ, σ, e).

Figures IA.1 to IA.5 offer visual evidence of how the targeted moments vary with the model parameters. To construct these figures, we first set all parameters (s, c, ρ, σ, e) at their estimated value. We then vary one of the parameters in partial equilibrium, that is, holding fixed w and Q. Importantly, the comparative statics that we report on these figures are direct simulation output: the relative smoothness of these plots gives us confidence in the precision of our numerical procedure, which we attribute to the dense grid for capital (about 300 points), debt (29 points), and productivity (51 points), as well as to a large number of simulated observations (1,000,000 firms over 10 years). See Section II of the Internet Appendix for details.

Figure IA.1 shows that β is nonmonotonic in s, the collateral parameter. Intuitively, for lower values of s, firms' investment decisions are constrained by collateral availability, so that an increase in s allows firms to extract more debt and investment capacity out of a \$1 increase in collateral value and β increases. For extreme values of s ($s \geq 1$), however, firms become unconstrained, and investment becomes independent of debt capacity, so that β goes to zero.

Table I

Elasticity of Moments with Respect to Parameters

This table reports the elasticity of simulated moments with respect to the estimated structural parameters. First, we start with the SMM estimate $\widehat{\Omega}$ of the parameters Ω. For each $k = 1, \ldots, 4$, we set $\omega_l = \hat{\omega}_l$ for all $l \neq k$, and vary the parameter ω_k around the estimated $\hat{\omega}_k$ to compute the elasticity of moments to parameters in the vicinity of the SMM estimate. For each moment m_n, we compute

$$\epsilon_{n,k} = \frac{\log m_n^+ - \log m_n^-}{\log \omega_k^+ - \log \omega_k^-} \approx \frac{\partial \log(\hat{m}_n)}{\partial \log(\hat{\omega}_k)},$$

where $\hat{m}_n$ is the n^{th} data moment, m_n^+ is the moment based on data simulated with parameter $\hat{\omega}_k^+$. m_n^- is the average of moments based on data simulated with parameter ω_k^-. For each parameter, we consider a 10-grid point scale as in Figures IA.1 to IA.5. Parameters ω_k^+ and ω_k^- are values just above and below the SMM estimate $\hat{\omega}_k$. For example, around the SMM estimate, a 1% increase in s is associated with a 1.2% decrease in the sensitivity of investment to real estate and a 1.1% increase in leverage.

	s.d. Δ log sales	s.d. Δ_5 log sales	Net Debt / Assets	$\beta(Inv, RE)$	Autocorr. Invest.	Equity Issues / Value-Added
Pledgeability s	0.066	0.071	1.1	1.2	−0.64	−0.23
Adj. cost c	−0.02	−0.013	0.029	−0.0058	0.31	−.071
Volatility σ	1.0	1.1	−0.7	0.7	0.26	3.8
Persistence ρ	0.81	2.1	−0.76	−2.5	5.5	13.0
Issuance cost e	−0.057	−0.13	−0.2	0.21	−0.72	−2.1

Around the SMM estimate (represented by a vertical line), β is a smooth and increasing function of s. Leverage is also smoothly increasing with the collateral parameter s. The first two panels of Figure IA.1 also show that an increase in s leads to an increase in output volatility: When the firm becomes less constrained, its capital stock responds more to productivity shocks.

The adjustment cost parameter c is mostly identified by the autocorrelation of investment (Figure IA.2). Large adjustment costs lead the firm to smooth investment over time, which leads to a large autocorrelation of investment. Larger adjustment costs to capital also lead to lower short-term output volatility. Similar to financing constraints, adjustment costs prevent firms from adjusting their capital stock in response to productivity shocks, making output less volatile. Figures IA.3 and IA.4 show that (i) the volatility of log productivity, σ, has a nearly linear effect on output volatility at all horizons, while (ii) the productivity persistence ρ mostly affects the long-term volatility of output. Taken together, these two observations are consistent with the idea that the ratio of one- to five-year output volatility allows us to identify the persistence parameter ρ. Note also that the persistence of productivity shocks has a sizable positive effect on the autocorrelation of investment. Firms can afford to delay their response to productivity shocks when these shocks are more persistent. Unsurprisingly, the cost of equity issuance e decreases monotonically with net equity issuance (Figure IA.5).

Table I provides the elasticities of each moment with respect to the estimated parameters—a simple transformation of the Jacobian matrix. More precisely, we compute for each moment m_n and each parameter ω_k, the following elasticity (Hennessy and Whited (2007)):

$$\epsilon_{n,k} = \frac{\log m_n^+ - \log m_n^-}{\log \omega_k^+ - \log \omega_k^-} \approx \frac{\partial \log(\hat{m}_n)}{\partial \log(\hat{\omega}_k)},$$

where $\hat{\omega}_k$ is the parameter value at the SMM estimate and $\hat{m}_n$ the corresponding value for moment n. The parameter $\hat{\omega}_k^+$ ($\hat{\omega}_k^-$) is the value located just above (below) on the grid used to plot Figures IA.1 to IA.5, where m_n^+ (m_n^-) is the corresponding simulated moment obtained using parameter $\hat{\omega}_k^+$ ($\hat{\omega}_k^-$), keeping the other parameters $\hat{\omega}_{k'}$ at their SMM estimate. Table I confirms the results in Figures IA.1 to IA.5.

E. Estimation Results

We report the results of the SMM estimation in Table II. Each column corresponds to a model specification. Column (1) assumes no equity issuance ($e = +\infty$) and no adjustment costs ($c = 0$), while column (2) allows for adjustment costs. Column (3) allows for both adjustment costs and equity issuance and constitutes our baseline specification. Column (4) of Table II contains the data.

Overall, the results show that the estimate of s, the pledgeability parameter, is quite robust to the introduction of adjustment costs and equity issuance. The estimate ranges from 0.20 to 0.25: each \$1 of capital provides about \$0.20 of debt capacity. This is reassuring, as it suggests that s is "pinned down" by the sensitivity of investment to real estate, and is not much affected by misspecification bias arising from the omission of real frictions or outside equity issuance. We explore misspecification bias more systematically in Section V.

The estimated persistence and volatility of productivity are stable across specifications. The estimated persistence ranges from 0.85 in our baseline model to 0.92 in the model with no adjustment cost and no equity issuances. The estimated volatility is 13%. Relative to the literature, we estimate relatively small adjustment costs, $c = 0.004$. This value is small compared to Cooper and Haltiwanger (2006), who find 0.049 for the same parameter. There are two reasons for this. First, their model does not have financing constraints. As can be seen from column (1), financing constraints already generate, without any adjustment cost, a positive level of persistence in investment that is not too far from the data moment. The other difference with Cooper and Haltiwanger (2006) is the presence of fixed costs in their model. Cooper and Haltiwanger (2006) use plant-level data, which exhibit lumpier investment than the annual firm-level data we use. The observed lumpiness in investment motivates the introduction of fixed costs in their analysis. However, fixed costs generate negative autocorrelation in investment rates. Cooper and Haltiwanger (2006), therefore, require a larger value for c to match the actual autocorrelation of investment. This is clear from their table IV: without fixed costs, the estimated level of quadratic adjustment costs c leads to an implausibly large autocorrelation.

Table II

Parameter Estimates (SMM)

This table reports the results of our SMM estimations. The estimation procedure is described in the text and in Section II of the Internet Appendix. Columns (1) to (3) correspond to SMMs using different models. Column (1) assumes no adjustment cost and infinite cost of equity issuance ($c = 0, e = +\infty$). Column (2) introduces adjustment costs but maintains $e = +\infty$. Column (3) further allows for a finite cost of equity issuance. For each of these estimations, Panel A shows the estimated parameters, along with standard errors (obtained via bootstrapping) in parentheses. Panel B shows the value of a set of moments, measured on simulated data (with 1,000,000 observations). Moments with a superscript "+" are those that are targeted in the estimation. The other moments are not targeted. The last column (labeled "data") reports the empirical moments.

Specification:	Model 1: $c = 0, e = +\infty$ (1)	Model 2: $c > 0, e = +\infty$ (2)	Model 3: $c > 0, e > 0$ (3)	Data (4)
Panel A: Estimated Parameters				
ρ	0.922	0.893	0.851	
	(0.013)	(0.020)	(0.017)	
σ	0.124	0.134	0.131	
	(0.007)	(0.004)	(0.003)	
s	0.196	0.216	0.250	
	(0.078)	(0.080)	(0.048)	
c	0	0.008	0.004	
		(0.003)	(0.002)	
e	$+\infty$	$+\infty$	0.091	
			(0.012)	
Panel B: Moments (Targeted Indicated with "+")				
SD one-year sales growth	0.327^{+}	0.327^{+}	0.327^{+}	0.327
SD five-year sales growth	0.913^{+}	0.912^{+}	0.912^{+}	0.912
Real-estate to assets	0.140^{+}	0.140^{+}	0.141^{+}	0.140
$\beta(Inv, RE)$	0.060^{+}	0.060^{+}	0.060^{+}	0.060
Autocorrelation of Inv.	0.041	0.165^{+}	0.165^{+}	0.165
Net equity issuance to value-added	0	0	0.026^{+}	0.026
$\beta(D, RE)$	0.059	0.053	0.070	0.060
Net debt to assets	0.030	0.090	0.171	0.098

Finally, we estimate a cost of equity issuance of \$0.09 per \$1 of new equity issued, which is in the ballpark of the existing empirical and structural literature (Hennessy and Whited (2007)).

Table II also analyzes two nontargeted moments. We first look at the effect of real estate collateral on net debt changes, estimated by using $\Delta debt_{it}$ as a dependent variable in Equation (1). Empirically, we estimate a significant coefficient of 0.06. All three specifications match this moment well (e.g., 0.070 in column (3)). We also consider the average net leverage ratio, which we define as net debt (dltt+dlc-che) divided by total assets (at). In our sample, the average net leverage is equal to 0.098. The estimated model in column (3) generates an average leverage ratio of 0.17, which is much larger than its empirical counterpart. We discuss leverage-based inference in greater detail in Section V.

In the remainder of the paper, we use the model in column (3) with adjustment costs and equity issuance costs as our baseline specification.

F. Determinants of Financing Constraints

We briefly discuss how firm characteristics covary with financing constraints in the cross-section of simulated data. To identify financially constrained firms, we first simulate data using our baseline estimated model (Table II, column (3)). For each simulated observation (i, t), we then compute its actual value and the value the firm would have if it started from the same state variables, but financing frictions were removed forever. We label a firm as constrained when its constrained value is less than 95% of its unconstrained value.

Panel A of Figure 1 shows the share of financially constrained firms across 20 equal-sized bins of productivity. We find that less productive firms are on average more constrained. Because productivity is mean-reverting, low productivity firms typically experience positive productivity shocks but have a low level of capital, which prevents them from borrowing and investing. Similarly, growing firms are more likely to be financially constrained (Panel B). In Panel C, we report the share of financially constrained firms across 20 equal-sized bins of log sales-to-capital (log MRPK). Log MRPK is closely related to distortions (e.g., Hsieh and Klenow (2009)). With Cobb-Douglas production and no friction, log MRPK measures marginal revenue productivity and should be equated across firms. Our setting involves several frictions (time to build, adjustment costs, and financing constraints) so that log MRPK is not equal to the user cost of capital. Instead, Panel C shows that high-MRPK firms tend to be more constrained on average since they have too little capital. The same intuition explains the result in Panel D, which shows that high-Tobin's Q firms are on average more likely to be financially constrained.

IV. General Equilibrium Analysis

To quantify the aggregate effects of financing frictions, we now embed our estimated firm dynamics model in general equilibrium and simulate counterfactual economies.

A. General Equilibrium Model

By clearing the goods and labor markets, the model endogenizes aggregate demand Q and the real wage w introduced in the model of Section II, equations (2) to (7).

Firms. A large number N of firms indexed by i produce intermediate inputs in quantity q_{it} at price p_{it}. Intermediates are combined into a CES-composite

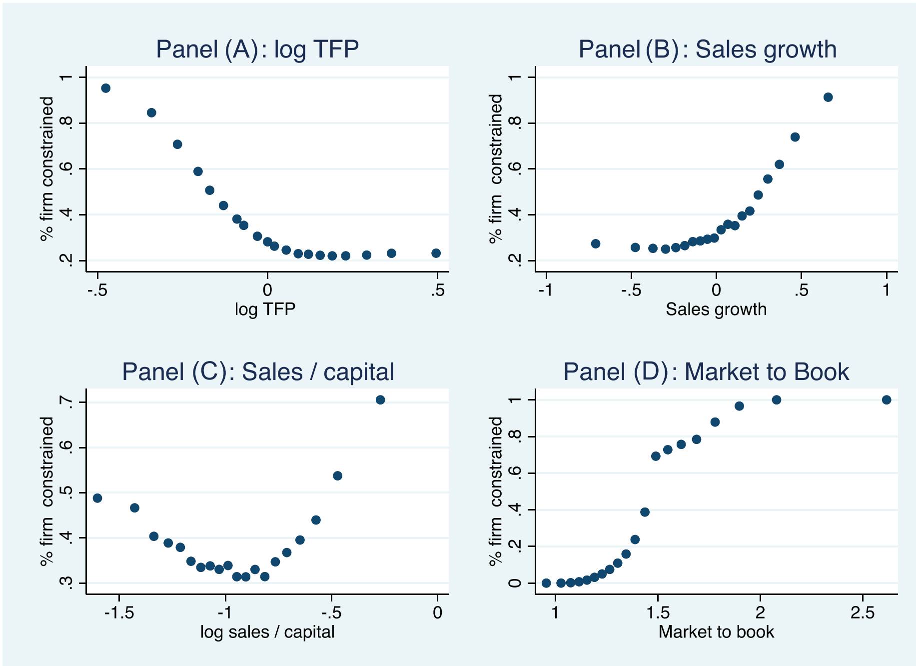

Figure 1. Financing constraints as a function of firm characteristics. This figure shows how the extent of financing constraints covaries with firm characteristics in the cross section of simulated firms. We simulate a data set of 1,000,000 firms over 215 years using parameters from our preferred specification (Table II, Panel A, column (3)). We remove the first 200 years to ensure firms are in steady state. For each characteristic x, we then sort firms into 20 equal-sized bins of x, and, for each bin, compute the average share of constrained firms. We label a firm-year as "constrained" if its market value is less than 95% of its unconstrained market value. Unconstrained market value is computed using the same set of state variables (z, k, b) at the beginning of the period and a model for which the cost of equity issuance e is set to zero. We use the following conditioning variables x: z (Panel A), $\log pq_t - \log pq_{t-1}$ (Panel B), $\log \frac{pq}{k}$ (Panel C), and $\frac{V}{k}$ (Panel D). (Color figure can be viewed at wileyonlinelibrary.com)

final good

$$Q_t = \left(\sum_{i=1}^{N} q_{it}^{\frac{\phi-1}{\phi}} \right)^{\frac{\phi}{\phi-1}}. \tag{8}$$

The final good is produced competitively. Thus, the final good price is given by $P_t = (\sum_i p_{it}^{1-\phi})^{\frac{1}{1-\phi}}$ and the demand for input i is given by $q_{it} = Q_t(\frac{p_{it}}{P_t})^{-\phi}$. We normalize $P_t = 1$ and derive the demand function in Equation (3).

Consumption and consumer behavior. The final good is used for (i) consumption, (ii) investment, and (iii) adjustment costs. The final good market equilibrium is thus given as

$$Q_t = C_t + Adj.\ Cost_t + I_t, \tag{9}$$

where C_t is aggregate consumption, $Adj.\ Cost_t = \sum_i \frac{c}{2} i_{it}^2 / k_{it}$ the sum of all adjustment costs, and $I_t = \sum_i i_{it}$ aggregate investment.

A representative consumer maximizes utility over consumption and labor,

$$U_s = \sum_{t \geq s} \beta^{t-s} u_t \text{ with } u_t = C_t - \bar{L}^{-\frac{1}{\epsilon}} \frac{L_t^{1+\frac{1}{\epsilon}}}{1+\frac{1}{\epsilon}}, \tag{10}$$

where L_t is aggregate hours worked, $\bar{L}$ a scaling constant, and ϵ the Frisch elasticity of labor supply. With quasi-linear preferences, the Hicksian, Marshallian, and Frisch labor supply elasticities are all equal to ϵ. Labor supply is a static decision given by

$$L_t^s = \bar{L} w_t^{\epsilon}. \tag{11}$$

Steady-state assumption and equilibrium definition. We assume that the economy is in steady state. In steady state, the consumption Euler equation ties the equilibrium interest rate r_t to the discount rate β, so the interest rate $r_t = 1/\beta - 1$ is pinned down throughout all counterfactuals. The "exogeneity" of r, a corollary to our steady-state assumption, holds for any additively separable utility function.

Intermediate good producers produce according to the Cobb-Douglas technology described in (2). The log productivity shocks z_{it} that they face have no aggregate component. Given our assumption that the number of firms is large, aggregate output Q and the wage w are constant over time. We are thus exactly in the case described in Section II and estimated in Section III.

Given the normalization $P_t = 1$, the equilibrium (Q, w) of this economy is defined by two equations, the labor market equilibrium and the final good aggregator:

$$\bar{L} w^{\epsilon} = \sum_{i=1}^{N} l^d\big((Q, w); z_{it}, k_{it}(Q, w)\big), \tag{12}$$

$$PQ = \sum_{i=1}^{N} p_{it} q\big((Q, w); z_{it}, k_{it}(Q, w)\big), \tag{13}$$

where $l^d(\cdot)$ is the numerically obtained labor demand function, which is a function of each firm state variable and aggregate equilibrium (Q, w). Similarly, $pq(\cdot)$ is the supply function, which, for each firm, associates state variables and macroeconomic conditions to its dollar sales. The equilibrium (Q, w) is the solution of these two conditions.

To solve for this equilibrium, we first simulate data from our estimated model using arbitrary starting values for (Q_0, w_0). Next, following Sraer and Thesmar (2018), we compute aggregate TFP and output using three sufficient statistics from the simulated data: the mean and variance of log MRPK (log $\frac{pq}{k}$ in the model), and the covariance of log MRPK with log firm-level TFP. Sraer and Thesmar (2018) show that this approach yields aggregate output and TFP in one iteration for any starting point (Q_0, w_0). We then solve for aggregate wage w, employment L, and total capital stock K.[13]

B. The Aggregate Effect of Financing Constraints

We now evaluate the aggregate effect of financing constraints based on our estimated model. Compared to the firm-level model, the macroeconomic model has a few additional parameters. Following Chetty (2012), we set the labor elasticity to $\epsilon = 0.50$. Labor elasticity does not affect TFP but affects our estimates of output losses from financing constraints.[14] We adjust $\bar{L}$ and the number of firms N so that the equilibrium parameter chosen for the estimation process ($Q = 1$ and $w = 0.03$) are actual equilibrium parameters when firm parameters are at the SMM estimate.

To measure the aggregate effect of financing constraints, we calculate aggregate TFP and output in log deviations from the "unconstrained" benchmark. The appropriate way to define the unconstrained benchmark in our model is to set equity issuance cost e to zero, rather than removing the collateral constraint. With $e = 0$, investment is unconstrained since equity is freely available to all firms. With no collateral constraint, firms would raise infinite debt for tax purposes. Our unconstrained benchmark thus corresponds to a model with free equity issuance and all other structural parameters—including the collateral constraint—unchanged. It also has the advantage of giving unconstrained firms the ability to benefit from the tax shield and lower their cost of capital, just like constrained ones. As we see below, as s increases, constrained and unconstrained economies behave more and more similarly.

Table III, Panel A, column (3) shows that, in our baseline model, financial frictions result in a large output loss of 7.1% relative to the unconstrained benchmark. The main channel for this output loss is the aggregate reduction in productive inputs: relative to the unconstrained benchmark, employment

[13] We use the following additional equations:

$$\log Q = \log TFP + \alpha \log K + (1-\alpha)\log L$$
$$\log Q = \log w + \log L - \log(1-\alpha)$$
$$\log L = \epsilon \log w.$$

[14] The effect, however, is modest. Chetty (2012) documents the range of existing estimates of ϵ. Summing up extensive and intensive margin elasticities, taking the 25th and 75th percentiles of the distribution, we obtain a range from 0.2 to 0.8. Over this range, output losses are from 6% to 9%.

Table III

Aggregate Effects of Collateral Constraints

This table reports results of the counterfactual analysis for different SMM parameter estimates. The general equilibrium analysis is described in Section IV and reported in Panel A. Columns (1) to (3) correspond to the three different models described in columns (1) to (3) of Table II: Column (1) assumes no adjustment cost ($c = 0$) and infinite cost of equity issuance ($e = +\infty$). Column (2) allows for adjustment cost but still assumes infinite cost of equity issuance. Column (3) also allows for finite cost of equity issues. Panel B implements the same methodology, but it holds the aggregate demand shifter Q constant while the wage w clears the labor market. Panel C holds both the aggregate demand shifter Q and wage w constant. Results in both panels are shown as log deviations from the constrained estimated model to the unconstrained benchmark. The unconstrained benchmark corresponds to an equilibrium in which firms face the same set of parameters as in the SMM estimate—reported in the same column, Table II, Panel A—but do not face a constraint on equity issuance ($e = 0$). In this unconstrained benchmark, investment reaches first best, but firms still benefit from the debt tax shield. For example, column (1) (no adjustment cost, no equity issuance) shows that the aggregate TFP loss compared to a benchmark without financing constraints is 3.1%.

Specification:	Model 1 $c = 0, e = +\infty$ (1)	Model 2 $c > 0, e = +\infty$ (2)	Model 3 $c > 0, e > 0$ (3)
	Panel A: General Equilibrium Results		
Δ log(TFP)	0.031	0.027	0.014
Δ log(Output)	0.151	0.120	0.071
Δ log(wage)	0.101	0.080	0.048
Δ log(L)	0.051	0.040	0.024
Δ log(K)	0.282	0.215	0.137
	Panel B: Partial Equilibrium Results, Holding Q Fixed Only		
Δ log(TFP)	0.012	0.012	0.005
Δ log(Output)	0.110	0.088	0.052
Δ log(wage)	0.073	0.059	0.035
Δ log(L)	0.037	0.029	0.017
Δ log(K)	0.240	0.185	0.117
	Panel C: Partial Equilibrium Results, Holding (Q, w) Fixed		
Δ log(TFP)	−0.040	−0.029	−0.020
Δ log(Output)	0.400	0.320	0.189
Δ log(wage)	-	-	-
Δ log(L)	0.400	0.320	0.189
Δ log(K)	0.531	0.417	0.254

is lower by 2.4% and capital by 13.7%. Financing frictions also lead to input misallocation, although this misallocation channel is quantitatively less important: aggregate TFP is lower by 1.4% in the estimated economy relative to the unconstrained benchmark. Taken together, these two channels reduce aggregate labor productivity, wages, and therefore labor supply, depressing employment. However, in this economy, the quantitatively important distortion induced by financing constraints is to prevent households from saving as much as they would want to (low capital stock), rather than allocating capital to the wrong firms (low TFP).

Table III, Panel A, also shows that adjustment costs tend to slightly attenuate the losses from financing constraints: the output loss is about 3 percentage points smaller in a model with adjustment costs (and no equity issuance, column (2)) than in a model with no adjustment cost and no equity issuance (column (1)). Similarly, TFP losses are 0.4 percentage points smaller in the presence of adjustment costs. In the presence of adjustment costs, firms smooth out investment, which becomes less responsive to productivity shocks. As a result, financing constraints bind less often. As such, real and financial frictions do interact nontrivially in our model, although quantitatively the role of this interaction is limited.

Panels B and C of Table III help quantify general equilibrium forces. Panel C provides the inference of a naive aggregator who takes the demand shifter Q and wage w as given. This leads to considerable overestimation of output effects of financing constraints (18.9% loss instead of 7.1%). This is because the labor supply elasticity is low (0.5). Unconstrained firms cannot hire many more workers as the labor market needs to clear. For the same reason, the effect on capital is also overestimated (25% instead of 13.7%). For employment, the effect is even larger (19% vs. 2%). In equilibrium, wages increase, which limits hiring and investing. The massive overestimation of K and L also explains why suppressing financing constraints actually *reduces* TFP. This is because the demand shifter is held constant at the firm level, which makes revenues less elastic to capital and labor—even though production is assumed to have constant returns to scale.

Figure 2 displays comparative statics for output and TFP losses with respect to the collateral parameter s. We fix all other parameters to their baseline estimates in column (3) of Table II. For 20 values of s around its estimated value, we solve the model and compute aggregate TFP and output losses from financing frictions. Figure 2 also reports the estimated collateral parameter s (vertical dark line), along with the 90% confidence band for this parameter (light blue bar). The precision of our estimate—a standard error of 0.045 for a point estimate of 0.25—implies that for values of s in the 90% confidence interval, output losses are between 6.5% and 8%, and TFP losses between 1.1% and 1.5%.

C. Additional Comparative Statics

Our model generates additional comparative static properties beyond the effect of s. Figure 3 further analyzes parameter effects on aggregate TFP (we show the effect on aggregate output in Figure IA.6 to save space). For the first three parameters, which are estimated, we report not only the point estimate, but also the edges of the confidence interval to give a sense of the precision of our quantitative exercise. The last comparative static exercise involves a calibrated parameter (the price elasticity ϕ) so there is no confidence interval. In all exercises, aside from the parameter that we vary, we set all

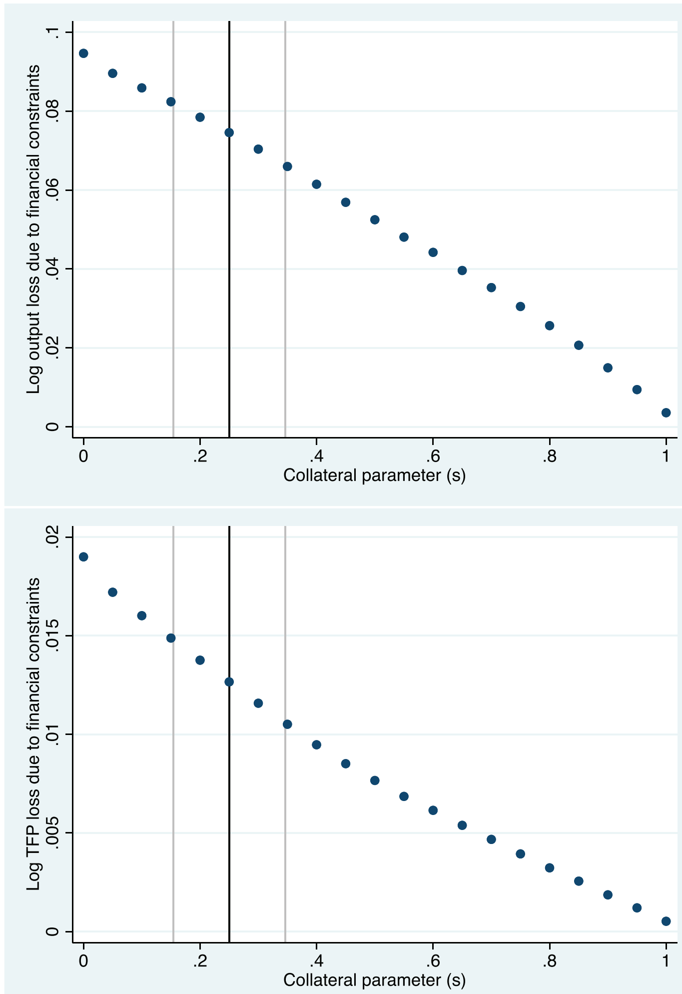

Figure 2. General equilibrium effect of pledgeability *s*. This figure reports the general equilibrium effect of changing the collateral parameter *s* from zero (the capital stock cannot be pledged as collateral) to one (100% of the capital stock can be pledged to lenders). For each value of *s*, we first compute aggregate output and TFP in a general equilibrium economy in which firms face a collateral parameter *s* and all other parameters are set to their estimate in Table II, Panel A, column (3). We then compute aggregate output and TFP in another general equilibrium economy where firms face the same collateral parameter *s* and all other parameters are set to their estimate in Table II, Panel A, column (3), except for the equity issuance parameter, which is now set to zero. This other economy corresponds to the unconstrained benchmark: in the absence of equity issuance costs, firms' investment will be first best. For each value of *s*, we then compute the log difference of output and TFP between these two economies. The vertical black line corresponds to the SMM estimate of *s* (0.25) and gray lines denote the limits of the 90% confidence interval. For example, when *s* increases from 0.1 to 0.6, the output loss relative to the unconstrained benchmark goes from 10% to 5%. (Color figure can be viewed at wileyonlinelibrary.com)

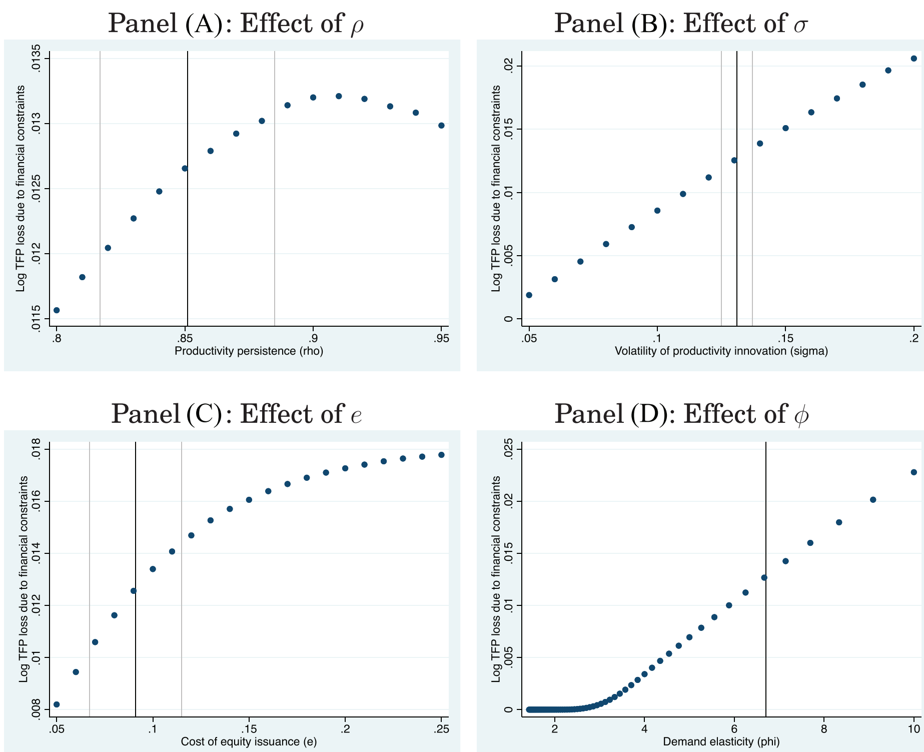

Figure 3. Total factor productivity: Additional comparative statics. This figure reports the effect of changing various parameters on the TFP loss of financing constraints. We vary productivity persistence in Panel A (ρ), productivity innovation volatility in Panel B (σ), equity issuance costs in Panel C (e), and price elasticity ϕ in Panel D. In Panels A, B, and C, the vertical black line correspond to the SMM estimates and the gray lines the borders of the 90% confidence interval. In Panel D, we do not report confidence intervals because ϕ is calibrated, not estimated. The values we span correspond to the range of values in Broda and Weinstein (2006).
(Color figure can be viewed at wileyonlinelibrary.com)

other parameters to their estimated values in our baseline specification (Table II, column (3)).

Panel A analyzes the effect of productivity persistence ρ. Existing papers emphasize that the persistence of productivity shocks should reduce distortions coming from financing frictions (Buera, Kaboski, and Shin (2011), Moll (2014)). This is because, in these papers, if shocks are persistent enough, firms can accumulate internal funds to free themselves of financing constraints. Our analysis in Panel A finds that this effect is dominated by an opposite force: when shocks are more persistent, the mismatch between productivity and capital stock lasts longer. Overall, the net effect of ρ on TFP is quantitatively small. A similar intuition holds in Panel B, where we explore the effect of productivity volatility on aggregate TFP: it is positive because it creates a more frequent mismatch, in the cross section of firms, between productivity and capital allocation.

Panel C describes the effect of the cost of equity issuance. Intuitively, the effect of equity issuance on TFP loss is sizable. Going from an issuance cost of 5% to 10% of proceeds (the estimate in Hennessy and Whited (2007) and close to our own estimate of 9.1%) yields an increase in TFP loss from 0.8% to 1.3%.

Finally, Panel D investigates the role of product market competition via the price elasticity ϕ. We explore the range presented in Broda and Weinstein (2006), table III, where ϕ goes from 1.5 (inelastic) to 14.1 (very elastic) product. Our chosen value (6.7) corresponds to the average value. We find a large sensitivity of TFP losses to ϕ. The aggregate loss of efficiency is negligible for $\phi < 3$: When demand is inelastic and competition weak, firms do not need to adjust their size much and thus are rarely constrained. When competition is strong, however, productivity shocks require movement of capital across firms at high frequencies, and financing constraints prevent this from happening.

V. Robustness to Misspecification

A key contribution of this paper is to ground the estimation of collateral constraints on a well-identified, reduced-form moment: the effect of collateral shocks on investment (β). The quantitative literature in structural corporate finance and macrofinance typically makes inference using a moment related to leverage, either at the firm level or in aggregate. This section explores how estimations based on leverage versus β differ in terms of robustness to misspecification.

A. Local Sensitivity to Leverage and β

We start with a generic formula for *local* misspecification bias. Let X be any statistic of interest (in our case, TFP or output loss from financing constraints). Assume that X is estimated by matching a misspecified model to a set of moments m (we look at two alternative sets of moments: one including leverage and one including β). Because the model is misspecified, the estimate $\hat{X}$ is biased. Andrews, Gentzkow, and Shapiro (2017) show that this misspecification bias is locally given by

$$\hat{X} - X = \underbrace{(\nabla X)' \times [-(J'WJ)^{-1}J'W]}_{\text{sensitivity}}(m^* - \hat{m}), \tag{14}$$

where $\hat{X}$ is the estimate (using the misspecified model) and X its true value, W is the SMM weighting matrix, J is the Jacobian matrix of the model, and ∇X is the gradient of X with respect to structural parameters, all computed at the SMM estimate (the term in ∇X is required because the statistic X is a function of structural parameters). In the last term in parentheses on the right-hand side, $\hat{m}$ are the empirical data moments, and m^* are the moments generated by the misspecified model but correct structural parameters. Thus, $(m^* - \hat{m})$ is the projected effect on moments of *removing* forces that are in the data but not in the model.

Andrews, Gentzkow, and Shapiro (2017) recommend computing the sensitivity matrix, which is easy since ∇X and J are known at the SMM estimate. The second term, the effect of removing nonmodeled forces on moments, is not knowable since it requires knowing what these forces are—unless one makes assumptions about what the real model is as we do below. The sensitivity matrix has an intuitive relation to the Jacobian matrix. Assume element (i, j) of $-(J'WJ)^{-1}J'W$, the inverse of the Jacobian, is large and positive (i.e., parameter i is strongly sensitive to moment j). Then if moment j is positively affected by unmodeled forces (i.e., $m_j^* < \hat{m}_j$), parameter i will be biased upward. Thus, moments with a high sensitivity of X should be relatively "pure" in the sense that they should be mostly driven by modeled forces. Moments with "large J" should therefore generate relatively little misspecification error.[15]

We compute the sensitivity of aggregate TFP and output losses from financial constraints. Table IV, Panel A, reports the (generalized) inverse of the Jacobian matrix. Panel B reports the gradient of output and TFP losses from financing constraints with respect to the parameters at their estimated value. Panel C constructs the product of the two, which corresponds to the sensitivity of output and TFP losses with respect to the estimated parameter. Table IV, Panel C, shows that the sensitivity of both TFP and output losses with respect to leverage is about double their sensitivity with respect to β. To illustrate this finding, consider a source of misspecification that leads to a mismeasurement of leverage equal to 0.1 and a mismeasurement of β equal to 0.1. Table IV, Panel C, implies that an estimation targeting leverage will result in a bias for TFP loss of 0.4 percentage points. In contrast, an estimation targeting β will result in a bias for TFP loss of only -0.2%.

Clearly, the sensitivity matrix is useful only if we have a sense of the extent to which data moments are affected by unmodeled forces. In other words, the above formula is useful only if we know how leverage and β would vary under particular sources of misspecification. To address this, in what follows we evaluate the effect of misspecification on these two moments using two approaches. In Section V.B, we consider sources of misspecification that arise purely from measurement issues. We assume that our model is correctly specified but that the moments used for estimation are mismeasured. In Section V.C, we assume instead that the moments are correctly measured but the model is misspecified. In both cases, we find that inference based on β is on average more robust to model misspecification that inference based on average leverage.

B. Moment Misspecification

In this section, we consider sources of misspecification that arise purely from measurement issues. We assume that our model is correctly specified, but that

[15] Note that this notion is close to the notion of asymptotic precision in estimation: assuming the model is correct, moments with "large J" contribute to making the SMM estimator more precise. This is because, in both cases (misspecification and estimation errors), the desirable property is that a parameter does not depend much on a moment.

Table IV

Calculating the Sensitivity of Baseline Estimates to Moments

This table reports the derivative of estimates of TFP loss and output loss with respect to moments, computed at the SMM estimate. This calculation follows Andrews, Gentzkow, and Shapiro (2017). Panel A reports the sensitivity matrix, defined as $\Lambda = -(J'WJ)^{-1}J'W$, where J is the Jacobian matrix estimated at the SMM estimate. Panel B reports ∇, the gradient of log TFP and output losses with respect to parameters at the SMM estimate. Panel C reports the sensitivity of these aggregate losses with respect to moments, $\nabla' \times \Lambda$.

Panel A: Sensitivity Matrix $\Lambda = -(J'WJ)^{-1}J'W$

	SD One-Year Sales Growth	*SD* Five-Year Sales Growth	Real Estate to Assets	Net Debt to Assets	β (Inv, RE)	Autocorr. of Inv.	Net Equity Iss. to Value-Added
ρ	2.54	−0.89	−0.08	0.24	−0.43	0.09	0.59
σ	−0.74	0.11	0.01	0.01	0.03	−0.07	0.11
s	0.19	−0.27	0.01	−1.04	−0.37	0.09	0.98
h	0.32	−0.29	−1.44	0.11	−0.08	−0.06	0.67
c	−0.14	0.03	0.01	−0.04	0.02	−0.07	0.14
e	0.75	−0.40	−0.02	0.22	−0.20	0.04	1.73

Panel B: Gradients of Log Output and TFP Losses

	ρ	σ	s	h	c	e
(∇ log Output loss)’	0.24	0.76	−0.07	0.00	−1.48	0.36
(∇ log TFP loss)’	0.02	0.12	−0.02	0.01	−0.03	0.10

Panel C: Gradient-Adjusted Sensitivity Matrix: $\nabla' \times \Lambda$

	SD One-Year Sales Growth	*SD* Five-Year Sales Growth	Real Estate to Assets	Net Debt to Assets	β (Inv, RE)	Autocorr. of Inv.	Net Equity Iss. to Value-Added
$(\nabla \log \text{ Output loss})' \times \Lambda$	0.51	−0.29	−0.03	0.27	−0.15	0.09	0.57
$(\nabla \log \text{ TFP loss})' \times \Lambda$	0.02	−0.04	−0.01	0.04	−0.02	0.00	0.17

the moments we target in estimation may be mismeasured. This approach is motivated by the vast corporate finance literature that has emphasized challenges to accurately measuring both the capital stock and debt. First, part of firms' actual capital is intangible and, therefore, missing from our measure of total book assets (Peters and Taylor (2017)). Second, part of the capital stock may be in the form of operating leases and therefore financed off balance sheet (Rampini and Eisfeldt (2009), Li, Whited, and Wu (2016)). Third, firms can borrow from their suppliers through accounts payables to finance total assets (Barrot (2016)). Fourth, the net book value of PPE may underestimate the true physical capital stock, as accounting depreciation is quite larger than the typical level of physical depreciation used in national accounts.

In Table V, we adjust our measure of debt, tangible assets, investment, and total assets to account for each of these challenges. Section IV of the Internet Appendix explains in detail the adjustments we make to obtain these measures. We then calculate average leverage in our sample and estimate β using these alternative definitions. Table V shows that average leverage is quite dispersed across these six alternatives: the baseline leverage is 0.093; when we account for intangible assets, leverage decreases to 0.072; when we include leases in both debt and assets, leverage goes up to 0.202. Across these alternatives, the standard deviation of the difference between our baseline leverage and the modified leverage is 0.058. At the same time, Table V also shows that β is more stable across these alternative specifications. Our baseline estimate for β is 0.06; when we include current assets in capital, β goes down to 0.028; when we include intangible capital in our measure of capital, β goes up to 0.083.[16] Across these alternatives, the standard deviation of the difference between our baseline β and the adjusted β is 0.021, about three times smaller than that for average leverage.[17]

Taken together, these results suggest a lower scope for misspecification when estimating TFP or output losses by targeting β: (i) the estimates for TFP and output losses are the times as sensitive to leverage than to β, and (ii) an exploration of misspecifications due to mismeasurement of debt and capital reveals that average leverage in these alternative specifications varies significantly more (relative to its baseline value) than β.

[16] Note that β remains statistically significant at the 1% confidence level across all of these specifications.

[17] Given the disproportionate importance of large firms in the macroeconomy, a natural question is whether our results are stable when weighting the moments of Table V by firm size. We show asset-weighted results in Table IA.II. Given the extreme skewness of the firm size distribution, weighted estimates are noisier but surprisingly stable compared to our core results. We focus on unweighted statistics in the main part of the paper, as these are the relevant moments on which to match our model for two reasons. First, these moments are more precisely estimated (they do not give a disproportionate weight to a few large firms). Second, the model has ex ante identical firms, so it is not designed to match the empirical firm size distribution. As a result, size-weighted moments generated by the model should not be expected to match size-weighted data moments.

Table V

Average Leverage Ratios and β Using Alternative Definition

Source: Compustat. The sample corresponds to the sample of firms in Chaney, Sraer, and Thesmar (2012). We calculate the average leverage ratio and estimate β under specific sources of misspecification. We use the following Compustat items: at is total assets; dltt is total long-term debt; dlc is debt in current liability; che is cash and short-term investment; ppent is property, plant, and equipment; capx is capital expenditures; xrd is R&D expense; xsga is selling, general and administrative expenses; act is total current assets; and ap is account payables. k_{int} is intangible capital, and $k_{\text{int}}^{\text{off-bs}}$ is its off-balance-sheet counterpart, from Peters and Taylor (2017); lease corresponds to leased operating capital and is calculated following an approach similar to Rampini and Eisfeldt (2009). For each firm-year, we compute l_{it}, the ratio of lagged one-year rental commitments (mrc1) to the rental cost of assets, which we measure as depreciation (dp) plus 10% of total assets (at). We trim observations for which this ratio is above one or below zero, and set it to zero when mrc1 is missing. We then multiply this ratio by total assets (at) to estimate the value of operating capital and implicit debt, assuming leverage being one for operating capital. To calculate PV(lease), we start from the next five years of commitments (mtr1-5), spread expected commitment (mrtca) equally over these five years, and calculate the present value of these commitments at a 10% discount rate. K corresponds to the capital stock calculated using a perpetual inventory method. For each firm, we take PPE (ppent) in the first fiscal year post-1981, depreciate it every year at 6% as in Midrigan and Xu (2014), and increase it with capital expenditures (capx) and decrease it with sales of property (sppe). Firm-clustered s.e. are between parentheses.

	Definition	D	Assets	K	I	Leverage = D/Assets	β
1	Standard	dltt+dlc-che	at	ppent	capx	0.093	0.060
						(0.007)	(0.007)
2	Intangible	dltt+dlc-che	at+$k_{\text{int}}^{\text{off-bs}}$	ppent+k_{int}	capx + xrd+.3×xsga	0.072	0.083
						(0.004)	(0.011)
3	Leasing 1	dltt+dlc-che+lease	at+lease	ppent+lease+lease-lease(t-1)	capx	0.202	0.065
						(0.006)	(0.010)
4	Leasing 2	dltt+dlc-che+PV(lease)	at	ppent	capx	0.130	0.060
						(0.008)	(0.007)
5	Account payables	dltt+dlc-che+ap	at	ppent+act	capx+act-act(t-1)	0.201	0.028
						(0.008)	(0.007)
6	Real depreciation	dltt+dlc-che	at+K-ppent	K	capx	0.074	0.070
						(0.006)	(0.012)
7	All adjustments	dltt+dlc-che +lease+ap	at+K-ppent +lease+$k_{\text{int}}^{\text{off-bs}}$	ppent+k_{int} +lease+act	capx+act-act(t-1) +lease-lease(t-1) +(1-τ)(xrd+.3×xsga)	0.184	0.037
						(0.005)	(0.022)

C. Model Misspecification

In the previous section, we consider misspecifications arising purely from measurement issues. In this section, we consider a different type of misspecification: we assume that the moments are correctly measured but that the model is misspecified. We consider several dimensions of model misspecification and investigate the robustness of inference based on β relative to inference based on leverage. Because we are interested in potentially large deviations from our baseline model, we depart from the local approximation approach of Andrews, Gentzkow, and Shapiro (2017) and instead rely on an MC approach.

We proceed in three steps: (i) simulate data under an auxiliary model, (ii) use simulated data to estimate our baseline (and hence misspecified) model, targeting either of the two moments, and (iii) compare the true TFP or output loss with the two misspecified estimates. Since this approach requires that we "specify the misspecification," our conclusions only apply to the particular set of alternative models we consider. We overcome this limitation by exploring 4,000 auxiliary models.

C.1. Auxiliary Models

We describe the class of auxiliary models that we use to simulate data. We expand the baseline model in six directions, each described by one additional parameter of varying intensity.

With the first group of three parameters, we investigate misspecifications that relate to how we model the capital stock. We first include intangible capital in the model. We assume that intangible capital (i) is a perfect complement to physical capital (i.e., the production function is Leontieff in intangible and tangible capital), (ii) cannot be collateralized, (iii) is unobserved by the econometrician, and (iv) depreciates at the same rate as tangible capital. We parameterize this modification of the baseline by $\mathcal{I}$, the fraction of intangible capital, which we allow to go from 0% to 40% (Peters and Taylor (2017)). As a result of these assumptions, the only modification to the model is that cash flows from investment are now given by

$$\frac{1}{1-\mathcal{I}}(-k_{it+1} + (1-\delta)k_{it} + \tau\delta k_{it}).$$

Our second specification considers the case in which tangible capital is itself mismeasured by a factor U. While in the model the tangible capital stock is k_{it}, the econometrician only observes $(1-U)k_{it}$. This factor is meant to account for operating leases, which represents productive capital that does not appear on the balance sheet. It also potentially accounts for the sizable discrepancy between accounting depreciation (about 15% in our data) and physical obsolescence of around 5% to 6% in the macrofinance literature (e.g., Midrigan and Xu (2014)). We restrict $U \in [0, 0.33]$, so that the econometrician may miss up to one-third of the true tangible capital stock. In our last specification, we allow the price of real estate p_t to be mismeasured. We assume that the log measurement error follows the same AR1 process as actual log prices but with an innovation volatility $\sigma_u \in [0; 0.03]$ equal to up to one-half of the actual innovation volatility of log prices (which is 0.06). This captures the fact that our price data may not be granular enough to capture the real variation in each firm's real estate prices. This will affect inference using our moment but not inference using leverage.

In the second group of three parameters, we allow for unmeasured sources of debt capacity/borrowing needs. First, note that our model may be misspecified in that some of the debt capacity may be unsecured and therefore less related to the amount of capital stock available. We assign firms a fixed amount of debt capacity d_0, so that their debt constraint is now

$$(1+r)d_{it+1} \leq d_0\bar{k} + s\big((1-\delta)k_{it+1} + \mathbb{E}(p_{t+1}|p_t) \times h\big),$$

where $\bar{k}$ is a scaling factor equal to the mean capital stock in the sample. We let $d_0 \in [0, 0.4]$. Second, we model the fact that operating leases offer an additional source of secured debt, which does not appear on the firm's balance sheet. We assume that observed net leverage is given by

$$lev_{it} = \frac{d_{it} + d_0\bar{k}}{k_{it}} - \kappa,$$

where κ captures the amount of debt capacity that the econometrician does not see when computing the classical net book leverage ratio. We allow $\kappa \in [-0.3, 0.3]$ to be negative to capture the notion that some of the existing debt capacity may be committed to noninvestment use (i.e., working capital finance). Third, we allow for mismeasurement in the effective tax rate that firms are facing. While the econometrician assumes $\tau = 0.33$, we assume that the true data-generating tax rate τ may deviate from the baseline. We cover $\tau \in [0.2, 0.4]$ in order to allow firms to face a lower effective tax rate than the statutory one, for instance, because of tax credits or tax loss carryforwards. This affects the attractiveness of debt and therefore capital structure.

We end up with an alternative data-generating process that, in addition to our baseline model, accounts for intangible capital ($\mathcal{I}$), tangible capital mismeasurement (U), measurement error of real estate value (σ_u), unobserved debt capacity (κ), unsecured debt capacity (d_0), and error in tax rate (τ). We now describe how we estimate misspecification biases in a cross section of 4,000 such simulated economies.

C.2. MC Approach

In this section, we assess the sensitivity to misspecification of inference based on β versus inference based on leverage. To do so, we start by constructing a large cross section of hypothetical economies generated under alternative models. Each of these alternative data-generating models is characterized by the five baseline structural parameters (ρ, σ, s, c, e) and six "misspecification" parameters $\Theta = (\mathcal{I}, U, \sigma_u, d_0, \kappa, \tau)$. We draw 4,000 such sets of 11 parameters uniformly. For each draw of these parameters, we follow the steps below:

(i) We solve the true model, simulate a data set, and calculate the log TFP and output losses from financing constraints under this true model. We do this by comparing the TFP/output of the true model, with the same aggregates under a version of this model with no equity issuance cost (setting $e = 0$).

(ii) Using the simulated data, we calculate all of the moments used in the estimation. We then use these moments to estimate the five parameters (ρ, σ, s, c, e) of our baseline model which assumes $\Theta = (0, 0, 0, 0, 0, 0.33)$ and is therefore misspecified. We perform two estimations:
   (a) We target the same moments as in Table II, column (3), which include, in particular, the reduced-form moment β. We then compute log TFP and output losses from financing constraints given the estimated parameters.
   (b) We target the same moments but replace the reduced-form moment β with average leverage. We again compute log TFP and output losses of financing constraints given these alternative estimated parameters.

(iii) For each of these estimated TFP and output losses, we calculate the misspecification error, that is, the difference between estimated TFP/output loss and the true TFP/output loss, which we can compute since we control the true data-generating process.

We obtain 4,000 misspecification biases for TFP loss and output loss when we target leverage and 4,000 misspecification biases for TFP losses and output loss when we target β instead. We normalize each one of these misspecification biases by their averages across all simulations to use a common scale (average TFP/output loss is 1.1% to 6.4% across simulations).

We describe the entire procedure in more detail in Section III in the Internet Appendix. In particular, we face a numerical challenge. The estimation of one model takes approximately one day, so we cannot solve 8,000 models (one targeting leverage, the other one targeting β) in a reasonable time. We overcome this numerical challenge by resorting to the following polynomial approximation. First, we simulate a large number of baseline economies using different parameters than the baseline estimate. For each of these economies, we simulate moments, including leverage and β. We then fit these moments with flexible polynomial functions of the parameters using OLS. We invert these polynomial functions and obtain functions that map sets of moments (one set with leverage, the other with β) into parameters. We show that this polynomial approximation is tight. These functions allow us to find parameters of the baseline model that best fit the data sets. On our sample of simulated economies, the R^2 obtained is of the order of 0.99. Finally, we go back to our 4,000 simulated economies (generated by the model with 11 parameters) and use the inverted polynomial functions to recover the estimates of the baseline model. The detailed procedure is described in Section III in the Internet Appendix.

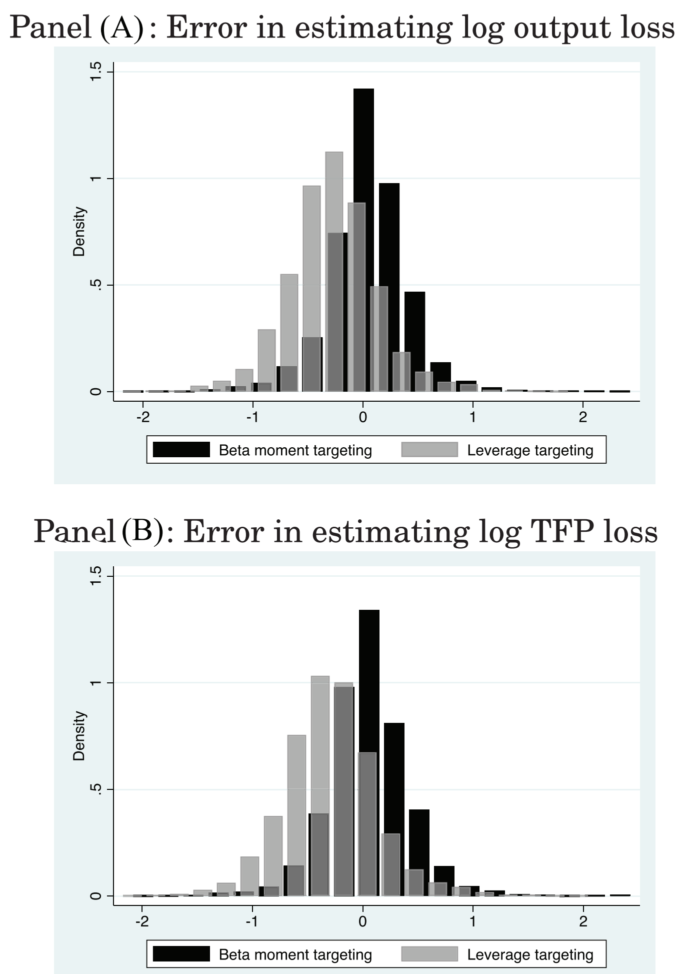

Figure 4. Error about aggregate estimates: Leverage versus β targeting. In this figure, we report the distribution of misspecification errors on TFP and output losses across 4,000 alternative models. We simulate data sets from 4,000 alternative models. Each alternative model corresponds to the baseline model augmented along six different dimensions as described in Section V.C.3. We estimate TFP/output losses using the baseline (misspecified) model on these 4,000 data sets using two approaches: one estimation targets leverage, while the other estimation targets the reduced-form moment β. For each alternative model, the difference between these estimated losses and the actual losses in the true model corresponds to the misspecification error in the estimation. We report the distribution of these errors for output loss in Panel A and for TFP loss in Panel B. (Color figure can be viewed at wileyonlinelibrary.com)

C.3. Results

Figure 4 reports the distribution of misspecification errors for output loss (Panel A) and TFP loss (Panel B) across all possible alternative models. While misspecification errors can be sizable under both approaches, they are on

Table VI

Estimation Error and Distance from Correct Specification

We simulate data sets from 4,000 alternative models. Each alternative model corresponds to the baseline model augmented along six different dimensions described in Section V.C.3. Six "misspecification" parameters control the degree of departure from the baseline model along these dimensions: $\Theta = (\mathcal{I}, U, \sigma_u, d_0, \kappa, \tau)$. We estimate the baseline (misspecified) model on these 4,000 data sets using two separate approaches: one estimation targets leverage, while another targets the reduced-form moment β. We then regress

$$\frac{\hat{X}_i - X_i}{\frac{1}{N}\sum_j X_j} = a + b\frac{\mathcal{I}_i}{\max_j \mathcal{I}_j} + c\frac{U_i}{\max_j U_j} + d\frac{\sigma_{u,i}}{\max_j \sigma_{u,j}} + e\frac{d_{0,i}}{\max_j d_{0,j}} + f\frac{\kappa_i}{\max_j \kappa_j} + g\frac{\tau_i - 0.33}{\max_j(\tau_j - 0.33)} + \epsilon_i$$

where X denotes estimated TFP/output losses and i indexes alternative models. Standard errors are omitted because they are irrelevant in this cross section of simulations, but the number is large enough to ensure smooth, linear relationships as shown in Figures IA.7 and IA.8. For example, when the fraction of intangible capital increases from 0 to 0.5 (maximum misspecification), the misspecification bias on TFP losses estimated by targeting leverage increases from zero (correctly specified) to 41% of the average TFP loss in the cross section.

Relative Error in Estimation of:	log TFP Loss		log Output Loss	
Misspecified SMM Targets:	β (1)	Leverage (2)	β (3)	Leverage (4)
Misspecification Parameters:				
Intangible capital share ($\mathcal{I}$)	−0.0056	−0.41	−0.0021	−0.39
Unobserved physical capital share (U)	−0.19	−0.34	−0.18	−0.33
Price measurement error (σ_u)	0.12	−0.0033	0.11	−0.0058
Unobserved debt capacity—need (d_0)	0.028	1.2	0.041	1.2
Fixed unsecured debt (κ)	0.098	−0.43	0.075	−0.42
Actual tax rate—33% ($\tau - 0.33$)	−0.73	−0.54	−0.68	−0.49
Constant	0.063	0.14	0.065	0.13
Observations	4,000	4,000	4,000	4,000
R^2	0.32	0.74	0.29	0.73

average close to zero when we target the reduced-form moment β. When targeting leverage, however, the distribution of errors is more dispersed and biased significantly downward: the average misspecification bias for both output and TFP loss is about 30% of the average true loss across all models (remember that both of these errors for TFP and output loss are both rescaled by their average across simulations).

To assess how misspecification errors depend on misspecified parameters, in Table VI, we report the results of the following regression across simulated economies i:

$$\frac{\hat{X}_i - X_i}{\frac{1}{N}\sum_j X_j} = a + b\frac{\mathcal{I}_i}{\max_j \mathcal{I}_j} + c\frac{U_i}{\max_j U_j} + d\frac{\sigma_{u,i}}{\max_j \sigma_{u,j}} + e\frac{d_{0,i}}{\max_j d_{0,j}} + f\frac{\kappa_i}{\max_j \kappa_j} + g\frac{\tau_i - 0.33}{\max_j(\tau_j - 0.33)} + \epsilon_i,$$

where $\frac{\hat{X}_i - X_i}{\frac{1}{N}\sum_j X_j}$ is the misspecification error for statistic X scaled by its average across simulations. There are four sets of results, depending on whether we are targeting β or leverage and whether we are calculating TFP or output loss. We do not report t-statistics since this is a simulated sample—all coefficients will end up significant with enough simulations. However, in Figures IA.7 to IA.8, we show that the number of simulations is large enough to ensure smooth dependence of misspecification errors as a function of the parameters governing the misspecification (Θ). These figures are essentially partial binscatter plots. To build them, we first regress log TFP and log output estimation errors on 10 deciles of each one of the six directions of mispecification ($9 \times 6 + 1$ dummies). For each direction, we then report on the y-axis the predicted value for each decile dummy, holding the other parameters at their average values. For the x-axis, we use the average of the parameter of the corresponding decile. The main observation in these figures is that the red line (error on β-based inference) is close to zero and relatively flat compared to the blue line (error on leverage-based inference).

Table VI confirms this result. Specifically, we find that for these particular sources of misspecification, inference based on β typically yields a more robust estimation than inference based on leverage. First, note that the constant is close to zero in both types of estimation. This is consistent with the idea that, if the model is correctly specified, the inference is correct under both reduced-form moment β- and leverage-targeting.

Second, the leverage-based inference is sensitive to the presence of misspecifications on debt—a problem that inference based on β does not face. The slope of misspecification errors with respect to misspecification arising from unsecured debt (unobserved debt capacity) is about four times larger (40 times larger) when the model is estimated by targeting leverage than by targeting β. These results are intuitive: estimation of β does not require any information on debt from firms' balance sheet. The signs of the slopes in Table VI are also intuitive. A higher share of unobserved debt capacity implies that observed leverage is too low relative to firms' true leverage. An estimation targeting leverage will thus underestimate true debt capacity, leading to an overestimation of losses generated by financing constraints (1.2 for both TFP and output losses). Again, since β is estimated without information on debt, the estimation based on β will not suffer from this issue.

Third, the leverage-based inference is also quite sensitive to sources of misspecification related to the capital stock. If the share of unobserved intangible assets increases by 1 percentage point, the average misspecification error for TFP loss decreases by 41 basis points (bps) if the model is estimated by targeting leverage; in contrast, if the model is estimated by targeting β, the average misspecification error for TFP loss drops by only by 0.5 bps, that is, 20 times less. The fact that the slope is negative is intuitive. Leverage-based inference overestimates the true leverage ratio as the true capital stock is larger than the measured assets. As a result, estimations targeting leverage underestimate the true extent of financing constraints. Similar conclusions hold qualitatively for other sources of misspecification, such as the existence of physical capital that is not measured in PPE.

Finally, we investigate two additional sources of misspecification: measurement error in real estate prices and that in the effective corporate income tax rate. Intuitively, measurement error in real estate prices affects inference based on β but not inference based on leverage. However, the misspecification bias for β-based estimates is modest in size. If half of the innovation in real estate prices was pure noise, log TFP losses would be overestimated by about 13% of the average TFP loss across simulations. The reason for this upward bias is intuitive: noise in real estate prices creates a downward bias in the investment regression, which our estimation wrongly attributes to a smaller collateral parameter s, leading to stronger financing constraints.

Misspecification related to firms' tax rate leads to similar biases for estimates based on β and estimates based on leverage. In both cases, if taxes are higher than their baseline value (33%), firms are more likely to sacrifice debt capacity to enjoy more tax shield. They are thus more constrained and removing constraints increases TFP and output more than under the baseline tax rate. This effect is independent of which moment is targeted in the estimation. In particular, inference based on β cannot address this issue.

VI. Conclusion

This paper quantifies the aggregate effects of a specific source of financing frictions, namely, collateral constraints. We build a standard dynamic general equilibrium model with heterogeneous firms and collateral constraints. The model is estimated by targeting a set of moments, including a reduced-form regression coefficient β, the observed sensitivity of investment to exogenous shocks to collateral values. The model suggests that financing constraints generate a 7% aggregate output loss relative to a frictionless benchmark. This output loss can be broken down into three channels: (i) 1.4% TFP loss (misallocation), (ii) 13.7% lower capital stock, and (iii) 2.4% employment loss. By using a well-identified estimate to quantify an equilibrium model of investment under financial frictions, our paper helps to bridge the gap between the reduced-form literature and the quantitative literature in macrofinance and corporate finance.

We show that our inference is more robust to specific sources of misspecification than standard inference targeting moments related to leverage. We start from the local approach in Andrews, Gentzkow, and Shapiro (2017) and show that estimated output and TFP losses are less sensitive to β than to leverage. We then consider nonlocal sources of misspecification: we simulate thousands of data sets under alternative models and estimate our baseline, misspecified model on these data sets. These MC simulations show that inference based on β is on average more robust to the sources of model misspecification we consider than inference based on leverage. We believe that the approach we develop to examine misspecification bias is one of the paper's main contributions. Our MC simulations are easy to implement and computationally inexpensive: they leverage a polynomial approximation of the relationship between moments and parameters, which we show is efficient in our context. These simulations provide robustness checks that are similar in spirit to what reduced-form researchers do in robustness analysis: they allow us to consider alternative forces in the model and verify that these forces affect the findings in limited ways.

While we focus on one particular moment in this paper, we believe that the approach that we highlight could be adopted more systematically to confront the large number of well-identified moments in the literature on financing constraints. For instance, a large literature in banking estimates how credit supply shocks affect corporate borrowing and investment (since at least Peek and Rosengren (2000)). Our methodology can be adapted by explicitly modeling bank behavior and targeting these moments, an approach followed by Herreño (2021). Identifying the effect of cash flow shocks on corporate investment would be another natural candidate. We believe that confronting all of these moments by structural models of firm dynamics with financing constraints represents an important agenda for future research.

Initial submission: May 24, 2018; Accepted: May 20, 2021
Editors: Stefan Nagel, Philip Bond, Amit Seru, and Wei Xiong

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Appendix S1: Internet Appendix.
Replication Code.